M22/5/MATHY/HP2/ENG/TZ1/XX/M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2022

Mathematics: applications and interpretation

Higher level

Paper 2

20 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.
A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.
R Marks awarded for clear **Reasoning**.
AG Answer given in the question and so no marks are awarded.
FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2.***

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is “first”.

1. (a) (i) $\frac{370+472}{2}$ ***(M1)***

Note: This ***(M1)*** can also be awarded for either a correct Q_3 or a correct Q_1 in part (a)(ii).

$Q_3 = 421$ ***A1***

(ii) their part (a)(i) – their Q_1 (clearly stated) ***(M1)***

$\text{IQR} = (421 - 318 =)\ 103$ ***A1***

[4 marks]

(b) $(Q_3 + 1.5(\text{IQR}) =)\ 421 + (1.5 \times 103)$ ***(M1)***

$= 575.5$

since $498 < 575.5$ ***R1***

Netherlands is not an outlier ***A1***

Note: The ***R1*** is dependent on the ***(M1)***. Do not award ***R0A1***.

[3 marks]

(c) not appropriate ("no" is sufficient) ***A1***

as r is too close to zero / too weak a correlation ***R1***

[2 marks]

(d) (i) 6 ***A1***

(ii) 4.5 ***A1***

(iii) 4.5 ***A1***

[3 marks]

(e) (i) $r_s = 0.683\ \ (0.682646\ldots)$ ***A2***

(ii) **EITHER**

there is a (positive) association between the population size and the score ***A1***

OR

there is a (positive) linear correlation between the ranks of the population size and the ranks of the scores (when compared with the PMCC of 0.249) ***A1***

[3 marks]

(f) lowering the top score by 20 does not change its rank so r_s is unchanged ***R1***

Note: Accept "this would not alter the rank" or "Netherlands still top rank" or similar. Condone any statement that clearly implies the ranks have not changed, for example: "The Netherlands still has the highest score."

[1 mark]

[Total 16 marks]

2. (a) (i) $\left(\frac{1}{2}\text{A}\hat{\text{O}}\text{B} =\right) \arccos\left(\frac{4}{4.5}\right) = 27.266...$ ***(M1)(A1)***

$\text{A}\hat{\text{O}}\text{B} = 54.532... \approx 54.5^\circ \ (0.951764... \approx 0.952 \text{ radians})$ ***A1***

Note: Other methods may be seen; award ***(M1)(A1)*** for use of a correct trigonometric method to find an appropriate angle and then ***A1*** for the correct answer.

(ii) a finding area of triangle

EITHER

area of triangle = $\frac{1}{2} \times 4.5^2 \times \sin(54.532...)$ ***(M1)***

Note: Award ***M1*** for correct substitution into formula.

$= 8.24621... \approx 8.25 \text{ m}^2$ ***(A1)***

OR

$\text{AB} = 2 \times \sqrt{4.5^2 - 4^2} = 4.1231\ldots$ ***(M1)***

area triangle $= \frac{4.1231\ldots \times 4}{2}$

$= 8.24621... \approx 8.25 \ (\text{m}^2)$ ***(A1)***

finding area of sector

EITHER

area of sector = $\frac{54.532...}{360} \times \pi \times 4.5^2$ ***(M1)***

$= 9.63661... \approx 9.64 \text{ m}^2$ ***(A1)***

OR

area of sector = $\frac{1}{2} \times 0.9517641.. \times 4.5^2$ ***(M1)***

$= 9.63661... \approx 9.64 \text{ m}^2$ ***(A1)***

THEN

area of segment = $9.63661... - 8.24621...$

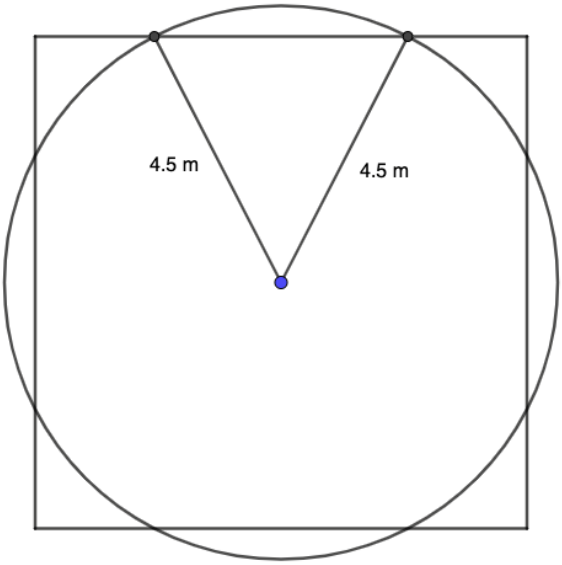
$= 1.39 \text{ m}^2 \ (1.39040...)$ ***A1***

[8 marks]

continued…

Question 2 continued

(b) **METHOD 1**



$\pi \times 4.5^2$ $(63.6172\ldots)$ ***(A1)***

$4 \times 1.39040...$ (5.56160) ***(A1)***

subtraction of four segments from area of circle ***(M1)***

$= 58.1 \text{ m}^2$ $(58.055...)$ ***A1***

METHOD 2

angle of sector = $90 - 54.532...$ $\left(\frac{\pi}{2} - 0.951764...\right)$ ***(A1)***

area of sector = $\frac{90 - 54.532...}{360} \times \pi \times 4.5^2$ $(= 6.26771...)$ ***(A1)***

area is made up of four triangles and four sectors ***(M1)***

total area = $(4 \times 8.2462...) + (4 \times 6.26771...)$

$= 58.1 \text{ m}^2$ $\left(58.055...\right)$ ***A1***

[4 marks]

(c) sketch of $\frac{\mathrm{d}V}{\mathrm{d}t}$ **OR** $\frac{\mathrm{d}V}{\mathrm{d}t} = 0.110363...$ **OR** attempt to find where $\frac{\mathrm{d}^2V}{\mathrm{d}t^2} = 0$ ***(M1)***

$t = 1$ hour ***A1***

[2 marks]

(d) recognizing $V = \int \frac{\mathrm{d}V}{\mathrm{d}t}\,\mathrm{d}t$ ***(M1)***

$\int_0^8 0.3te^{-t}\mathrm{d}t$ ***(A1)***

volume eaten is $0.299... \text{ m}^3$ $(0.299094...)$ ***A1***

[3 marks]

[Total 17 marks]

3. (a) quota ***A1***

[1 mark]

(b) (i) $27.125 \approx 27.1$ ***(M1)A1***

(ii) $8.29815\ldots \approx 8.30$ ***A1***

[3 marks]

(c) (let μ be the national mean)

H_0: $\mu = 25.2$
H_1: $\mu > 25.2$ ***A1***

Note: Accept hypotheses in words if they are clearly expressed and '**population** mean' or 'school mean' is referred to. Do not accept H_0: $\mu = \mu_0$ unless μ_0 is explicitly defined as "national standard mark" or given as 25.2.

recognizing t-test ***(M1)***
p-value $= 0.279391\ldots$ ***A1***

$0.279391\ldots > 0.05$ ***R1***

Note: The ***R1*** mark is for the comparison of their p-value with 0.05.

insufficient evidence to reject the null hypothesis (that the mean for the school is 25.2) ***A1***

Note: Award the final ***A1*** only if the **null** hypothesis is also correct (e.g. $\mu_0 = 25.2$ or (population) mean $= 25.2$) and the conclusion is consistent with both the direction of the inequality and the alternative hypothesis.

[5 marks]

(d) **EITHER**
the sampling process is not random ***R1***

For example:
the school asked for volunteers
the students were selected from a single class

OR
the quota might not be representative of the student population ***R1***

For example:
the school may have only 4 boys and 400 girls.

Note: Do not accept 'the sample is too small'.

[1 mark]

(e) (i) $(28.1 \times 2 + 20 =)\ 76.2$ ***A1***

(ii) 8.4×2 ***(A1)***
$= 16.8$ ***A1***

[3 marks]
[Total 13 marks]

4. (a) (i) $y = \frac{\mathrm{d}x}{\mathrm{d}t} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}t} + 5\frac{\mathrm{d}x}{\mathrm{d}t} + 6x = 0$ **OR** $\frac{\mathrm{d}y}{\mathrm{d}t} + 5y + 6x = 0$ ***M1***

Note: Award ***M1*** for substituting $\frac{\mathrm{d}y}{\mathrm{d}t}$ for $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$.

$$\begin{pmatrix} \frac{\mathrm{d}x}{\mathrm{d}t} \\ \frac{\mathrm{d}y}{\mathrm{d}t} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -6 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$ ***AG***

(ii) $\det \begin{pmatrix} -\lambda & 1 \\ -6 & -5-\lambda \end{pmatrix} = 0$ ***(M1)***

Note: Award ***M1*** for an attempt to find eigenvalues. Any indication that $\det(\boldsymbol{M} - \lambda \boldsymbol{I}) = 0$ has been used is sufficient for the ***(M1)***.

$-\lambda(-5-\lambda) + 6 = 0$ **OR** $\lambda^2 + 5\lambda + 6 = 0$ ***(A1)***

$\lambda = -2, -3$ ***A1***

(iii) (on a phase portrait the particle approaches $(0, 0)$ as t increases so long term velocity (y) is)

0 ***A1***

Note: Only award ***A1*** for 0 if both eigenvalues in part (a)(ii) are negative. If at least one is positive accept an answer of '*no limit*' or '*infinity*', or in the case of one positive and one negative also accept '*no limit or 0 (depending on initial conditions)*'.

[5 marks]

(b) (i) $y = \frac{\mathrm{d}x}{\mathrm{d}t}$

$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = \frac{\mathrm{d}y}{\mathrm{d}t}$ ***(A1)***

$\frac{\mathrm{d}y}{\mathrm{d}t} + 5y + 6x = 3t + 4$ ***A1***

(ii) recognition that $h = 0.1$ in any recurrence formula ***(M1)***

$(t_{n+1} = t_n + 0.1)$

$x_{n+1} = x_n + 0.1y_n$ ***(A1)***

$y_{n+1} = y_n + 0.1(3t_n + 4 - 5y_n - 6x_n)$ ***(A1)***

(when $t = 1$,) $x = 0.64402... \approx 0.644$ m ***A2***

(iii) recognizing that y is the velocity

$0.5\ \mathrm{m\,s^{-1}}$ ***A1***

[8 marks]

[Total 13 marks]

5. (a) (let T be the number of passengers who arrive)

$\left(\mathrm{P}(T > 72) =\right)\ \mathrm{P}(T \geq 73)$ **OR** $1 - \mathrm{P}(T \leq 72)$ ***(A1)***

$T \sim \mathrm{B}(74,\ 0.9)$ **OR** $n = 74$ ***(M1)***

$= 0.00379\ \ (0.00379124...)$ ***A1***

Note: Using the distribution $\mathrm{B}(74,\ 0.1)$, to work with the 10% that do not arrive for the flight, here and throughout this question, is a valid approach.

[3 marks]

(b) (i) 72×0.9 ***(M1)***

64.8 ***A1***

(ii) $n \times 0.9 = 72$ ***(M1)***

80 ***A1***

[4 marks]

(c) **METHOD 1**

EITHER

when selling 74 tickets

	$T \leq 72$	$T = 73$	$T = 74$
Income minus compensation (I)	11100	10800	10500
Probability	0.9962…	0.003380…	0.0004110…

top row ***A1A1***

bottom row ***A1A1***

Note: Award ***A1A1*** for **each** row correct. Award ***A1*** for one correct entry and ***A1*** for the remaining entries correct.

$\mathrm{E}(I) = 11100 \times 0.9962... + 10800 \times 0.00338... + 10500 \times 0.000411 \approx 11099$ ***(M1)A1***

continued…

Question 5 continued

OR

income is $74 \times 150 = 11100$ ***(A1)***

expected compensation is

$0.003380... \times 300 + 0.0004110... \times 600 \ (= 1.26070...)$ ***(M1)A1A1***

Note: The ***(M1)*** is for an attempt to work out expected compensation by multiplying a probability for tickets sold by either 300 or 600.

expected income when selling 74 tickets is $11100 - 1.26070\ldots$ ***(M1)***

Note: Award ***(M1)*** for subtracting their expected compensation from 11100.

$= 11098.73..\ (= \$11099)$ ***A1***

THEN

income for 72 tickets $= 72 \times 150 = 10800$ ***(A1)***

so expected gain $\approx 11099 - 10800 = \$299$ ***A1***

METHOD 2

for 74 tickets sold, let C be the compensation paid out

$P(T = 73) = 0.00338014\ldots,\ P(T = 74) = 0.000411098\ldots$ ***A1A1***

$E(C) = 0.003380... \times 300 + 0.0004110... \times 600 \ (= 1.26070...)$ ***(M1)A1A1***

extra expected revenue $= 300 - 1.01404... - 0.246658\ldots \ (300 - 1.26070\ldots)$

(A1)(M1)

Note: Award ***A1*** for the 300 and ***M1*** for the subtraction.

$= \$299$ (to the nearest dollar) ***A1***

METHOD 3

let D be the change in income when selling 74 tickets.

	$T \leq 72$	$T = 73$	$T = 74$
Change in income	300	0	−300

(A1)(A1)

Note: Award ***A1*** for one error, however award ***A1A1*** if there is no explicit mention that $T = 73$ would result in $D = 0$ and the other two are correct.

$P(T \leq 73) = 0.9962...,\ P(T = 74) = 0.000411098\ldots$ ***A1A1***

$E(D) = 300 \times 0.9962... + 0 \times 0.003380... - 300 \times 0.0004110$ ***(M1)A1A1***

$= \$299$ ***A1***

[8 marks]

[Total 15 marks]

6. (a) (i) $y = x^{\frac{1}{2}}$ ***(M1)***

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}x^{-\frac{1}{2}}$ ***A1***

(ii) gradient at $x = 0.16$ is $\frac{1}{2} \times \frac{1}{\sqrt{0.16}}$ ***M1***

$= 1.25$

EITHER

$y - 0.4 = 1.25(x - 0.16)$ ***M1***

OR

$0.4 = 1.25(0.16) + b$ ***M1***

Note: Do not allow working backwards from the given answer.

THEN

hence $y = 1.25x + 0.2$ ***AG***

[4 marks]

(b) $p = 0.45,\ q = 0.4125$ (or 0.413) (accept "$(0.45,\ 0.4125)$") ***A1A1***

[2 marks]

(c) (i) $(h(x) =)\ \frac{1}{2}\sqrt{2(x - 0.2)}$ ***A2***

Note: Award ***A1*** if only two correct transformations are seen.

(ii) $(a =)\ 0.28$ ***A1***

(iii) **EITHER**

Correct substitution of their part (b) (or $(0.28,\ 0.2)$) into the given expression ***(M1)***

OR

$\frac{1}{2}(1.25 \times 2(x - 0.2) + 0.2)$ ***(M1)***

Note: Award ***M1*** for transforming the equivalent expression for f correctly.

THEN

$(b =)\ -0.15$ ***A1***

[5 marks]

continued…

Question 6 continued

(d) (i) recognizing need to add two integrals ***(M1)***

$\int_0^{0.16} \sqrt{x}\ \mathrm{d}x + \int_{0.16}^{0.5} (1.25x + 0.2)\ \mathrm{d}x$ ***(A1)***

Note: The second integral could be replaced by the formula for the area of a trapezoid $\frac{1}{2} \times 0.34(0.4 + 0.825)$.

$0.251\ \mathrm{m}^2\ (0.250916\ldots)$ ***A1***

(ii) **EITHER**

area of trapezoid $\frac{1}{2} \times 0.05(0.4125 + 0.825) = 0.0309375$ ***(M1)(A1)***

OR

$\int_{0.45}^{0.5} (8.25x - 3.3)\,\mathrm{d}x = 0.0309375$ ***(M1)(A1)***

Note: If the rounded answer of 0.413 from part (b) is used, the integral is $\int_{0.45}^{0.5} (8.24x - 3.295)\,\mathrm{d}x = 0.03095$ which would be awarded ***(M1)(A1)***.

THEN

shaded area $= 0.250916\ldots - 0.0627292 - 0.0309375$ ***(M1)***

Note: Award ***(M1)*** for the subtraction of both $0.0627292\ldots$ and their area for the trapezoid from their answer to (a)(i).

$= 0.157\ \mathrm{m}^2\ (0.15725)$ ***A1***

[7 marks]

[Total 18 marks]

7. (a) (i) $\boldsymbol{P}\begin{pmatrix}0\\0\end{pmatrix}+\boldsymbol{q}=\begin{pmatrix}0\\1\end{pmatrix}$ ***(M1)***

$\boldsymbol{q}=\begin{pmatrix}0\\1\end{pmatrix}$ ***A1***

(ii) **EITHER**

$\boldsymbol{P}\begin{pmatrix}1\\0\end{pmatrix}+\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}\frac{\sqrt{3}}{4}\\\frac{3}{4}\end{pmatrix}$ ***M1***

hence $\boldsymbol{P}\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}\frac{\sqrt{3}}{4}\\-\frac{1}{4}\end{pmatrix}$ ***A1***

$\boldsymbol{P}\begin{pmatrix}0\\1\end{pmatrix}+\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}\frac{1}{4}\\1+\frac{\sqrt{3}}{4}\end{pmatrix}$ ***M1***

hence $\boldsymbol{P}\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}\frac{1}{4}\\\frac{\sqrt{3}}{4}\end{pmatrix}$ ***A1***

continued…

Question 7 continued

OR

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}+\begin{pmatrix} 0 \\ 1 \end{pmatrix}=\begin{pmatrix} \frac{\sqrt{3}}{4} \\ \frac{3}{4} \end{pmatrix}$$ ***M1***

hence $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}=\begin{pmatrix} \frac{\sqrt{3}}{4} \\ -\frac{1}{4} \end{pmatrix}$ ***A1***

$$\begin{pmatrix} a \\ c \end{pmatrix}=\begin{pmatrix} \frac{\sqrt{3}}{4} \\ -\frac{1}{4} \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}+\begin{pmatrix} 0 \\ 1 \end{pmatrix}=\begin{pmatrix} \frac{1}{4} \\ 1+\frac{\sqrt{3}}{4} \end{pmatrix}$$ ***M1***

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}=\begin{pmatrix} \frac{1}{4} \\ \frac{\sqrt{3}}{4} \end{pmatrix}$$ ***A1***

$$\begin{pmatrix} b \\ d \end{pmatrix}=\begin{pmatrix} \frac{1}{4} \\ \frac{\sqrt{3}}{4} \end{pmatrix}$$

THEN

$$\Rightarrow \boldsymbol{P}=\begin{pmatrix} \frac{\sqrt{3}}{4} & \frac{1}{4} \\ -\frac{1}{4} & \frac{\sqrt{3}}{4} \end{pmatrix}$$ ***AG***

[6 marks]

(b) $\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$ ***A1***

[1 mark]

continued…

Question 7 continued

(c) (i) **EITHER**

$$\boldsymbol{S}^{-1} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \quad \textit{(A1)}$$

$$\boldsymbol{R} = \boldsymbol{P}\boldsymbol{S}^{-1} \quad \textit{(M1)}$$

Note: The ***M1*** is for an attempt at rearranging the matrix equation. Award even if the order of the product is reversed.

$$\boldsymbol{R} = \begin{pmatrix} \frac{\sqrt{3}}{4} & \frac{1}{4} \\ -\frac{1}{4} & \frac{\sqrt{3}}{4} \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \quad \textit{(A1)}$$

OR

$$\begin{pmatrix} \frac{\sqrt{3}}{4} & \frac{1}{4} \\ -\frac{1}{4} & \frac{\sqrt{3}}{4} \end{pmatrix} = \boldsymbol{R} \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}$$

let $\boldsymbol{R} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

attempt to solve a system of equations ***M1***

$$\frac{\sqrt{3}}{4} = 0.5a\,, \quad \frac{1}{4} = 0.5b$$

$$-\frac{1}{4} = 0.5c\,, \quad \frac{\sqrt{3}}{4} = 0.5d \quad \textit{A2}$$

Note: Award ***A1*** for two correct equations, ***A2*** for all four equations correct.

THEN

$$\boldsymbol{R} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \textbf{ OR } \begin{pmatrix} 0.866 & 0.5 \\ -0.5 & 0.866 \end{pmatrix} \textbf{ OR } \left(\begin{pmatrix} 0.866025... & 0.5 \\ -0.5 & 0.866025... \end{pmatrix} \right) \quad \textit{A1}$$

Note: The correct answer can be obtained from reversing the matrices, so do not award if incorrect product seen. If the given answer is obtained from the product $\boldsymbol{R} = \boldsymbol{S}^{-1}\boldsymbol{P}$, award ***(A1)(M1)(A0)A0***.

continued…

Question 7 continued

(ii) clockwise ***A1***
arccosine or arcsine of value in matrix seen ***(M1)***
30° ***A1***

Note: Both ***A1*** marks are dependent on the answer to part (c)(i) and should only be awarded for a valid rotation matrix.

[7 marks]

(d) **METHOD 1**

(i) $\begin{pmatrix} a \\ b \end{pmatrix} = \boldsymbol{P}\begin{pmatrix} a \\ b \end{pmatrix} + \boldsymbol{q}$ ***A1***

(ii) solving $\begin{pmatrix} a \\ b \end{pmatrix} = \boldsymbol{P}\begin{pmatrix} a \\ b \end{pmatrix} + \boldsymbol{q}$ using simultaneous equations or $\boldsymbol{a} = (\boldsymbol{I} - \boldsymbol{P})^{-1}\boldsymbol{q}$
(M1)

$a = 0.651\ (0.651084...),\ b = 1.48\ \ (1.47662...)$ ***A1A1***

$\left(a = \dfrac{5 + 2\sqrt{3}}{13},\ b = \dfrac{14 + 3\sqrt{3}}{13} \right)$

METHOD 2

(i) $\begin{pmatrix} x' \\ y' \end{pmatrix} = \boldsymbol{P}\begin{pmatrix} x - a \\ y - b \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$ ***A1***

Note: Accept substitution of x and y (and x' and y') with particular points given in the question.

(ii) $\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \boldsymbol{P}\begin{pmatrix} 0 - a \\ 0 - b \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$ ***(M1)***

Note: This line, with any of the points substituted, may be seen in part (d)(i) and if so the ***M1*** can be awarded there.

$\begin{pmatrix} 0 \\ 1 \end{pmatrix} = (\boldsymbol{I} - \boldsymbol{P})\begin{pmatrix} a \\ b \end{pmatrix}$
$a = 0.651084...,\ b = 1.47662...$ ***A1A1***

$\left(a = \dfrac{5 + 2\sqrt{3}}{13},\ b = \dfrac{14 + 3\sqrt{3}}{13} \right)$

[4 marks]
[Total 18 marks]